

Acoustic metamaterials: An overview

Introduced to the scientific community by Sir John Pendry in the late 90s [1] the term “metamaterial” is used to denote an artificial material engineered to have a structure and properties not found in natural materials. Among these properties are negative refraction (waves bending in the opposite direction to what is expected in conventional materials), superlensing (enabling resolution of details smaller than the wavelength of the phenomena being influenced) and perfect absorption. Their unique properties open doors to innovative implementations across a range of fields and have garnered an interest for various commercial applications.

A specific subset of metamaterials which exhibit unusual interactions with acoustic phenomena are acoustic metamaterials. By tailoring their internal geometry and periodicity at scales comparable to or smaller than the wavelength of sound, acoustic metamaterials can control the propagation, absorption, and diffraction of sound waves with high precision, leading to advanced applications in noise control, medical imaging, architectural acoustics and many other fields and contexts. In spite of advancements during the past two decades, the field of acoustic metamaterials faces several challenges, among them the inherent complexity of designing and applying them in real-world scenarios - something further research seeks to address by developing more robust fabrication techniques, exploring novel material compositions, and optimizing the performance of acoustic metamaterials for specific applications.

This paper aims to give a broad overview of the topic, sketching out the historical development of metamaterials, their physical properties and interaction with acoustic phenomena, as well as their various applications in the real world and possible future development. The field of acoustic metamaterials is relatively underexplored, and may present rich challenges and opportunities to explore in acoustics.

1. Definition and origin

The term metamaterial is not very precisely defined, but a good working definition is: a material with ‘on-demand’ effective properties without the constraints imposed by what nature provides. Metamaterials derive their properties not from the properties of the base materials, but from their specifically designed structures which are arranged in repeating patterns. The arrangement, size and orientation of these patterns enables metamaterials to influence physical phenomena in a manner unusual to common materials (their name being derived from the Greek “*meta*” - beyond, and Latin “*materia*” - matter or material). The development of acoustic metamaterials was consequently derived from research on general metamaterials: Sir John Pendry developed the basic elements of metamaterials in the late 90s [2] which were soon developed into negative index materials (a metamaterial whose refractive index for an electromagnetic wave has a negative value over some frequency range) during the early 2000s.

Controlling acoustic waves has generally presented a challenge to scientists and engineers due to their longer wavelength and weaker attenuation compared to other types of waves (such as electromagnetic waves), as well as their strong diffraction and penetration abilities [3]. On one hand

these abilities enable acoustic waves to be utilized in areas such as ultrasound imaging, but before the advent of acoustic metamaterials prevented them from being utilized in a more “directed” or attenuated manner. The development of sonic crystals - structures which exhibit spectral gaps two orders of magnitude smaller than the wavelength of sound - created new methods of controlling sound waves. These crystals are also sometimes referred to as phononic crystals, echoing the development of photonic crystals in optics - though this term is more often reserved for materials formed by periodic variation of the acoustic properties of the material (i.e., elasticity and mass).

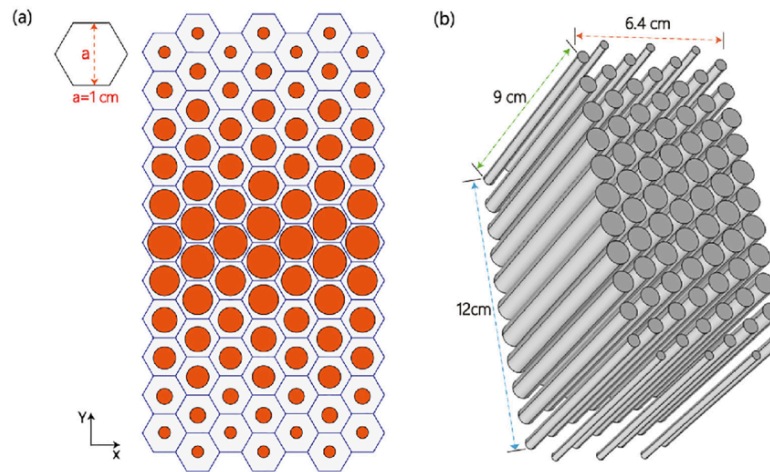


Image 1.a - schematic sample of a phononic crystal structure

The size and geometry of spectral gaps in a sonic crystal determines the transmission of sound waves at a certain frequency. One early research example from 2000, fabricated a structure with high-density solid lead balls as the core, one centimeter in size and coated with a 2.5-mm layer of rubber silicone arranged in an $8 \times 8 \times 8$ cube crystal lattice structure. Transmission was measured as a function of frequency from 250 to 1600 Hz for a four-layer sonic crystal. A two-centimeter slab absorbed sound that normally would require a much thicker material, at 400 Hz. A drop in amplitude was observed at 400 and 1100 Hz [4]. The coated spheres absorbed oscillations and created the observed frequency gap - the sound energy was absorbed exponentially as the thickness of the structure increased.

Similarly to sonic crystals, phononic crystals are materials designed to control and manipulate phonons (quasiparticles representing quantized vibrations in a solid), but they focus on affecting the propagation of mechanical vibrations at the microscopic level. One of their main properties is the possibility of having a phononic band gap. A phononic crystal with phononic band gap prevents phonons of selected ranges of frequencies from being transmitted through the material. Isaac Newton anticipated the development of phononic crystals by imagining the propagation of sound waves through the air in the same way an elastic wave would propagate along a lattice of so called “point masses” (or pointlike mass, a theoretical object with nonzero mass but infinitesimal volume) connected by springs with an elastic force constant. The size and arrangement of the elements comprising the phononic crystal determines the position of the band gap in a sound frequency - the width of the gap is generally related to the difference in the speed of sound (due to impedance differences). Phononic crystals are relatively effective at reducing low-frequency noise

due to their locally resonant systems acting as spatial frequency filters [5]. Given that electromagnetic and acoustic waves share parallel concepts of refractive index and crystal structure they can both be mathematically described by wave equations - thus the simplest realization of an acoustic metamaterial would involve the propagation of a pressure (sound) wave through a structure with a periodically modified refractive index in one dimension. In that case transfer matrices (a method used in optics) can be used to predict and analyze the behavior of a sound wave in a crystal 'slab' or 'stack'.

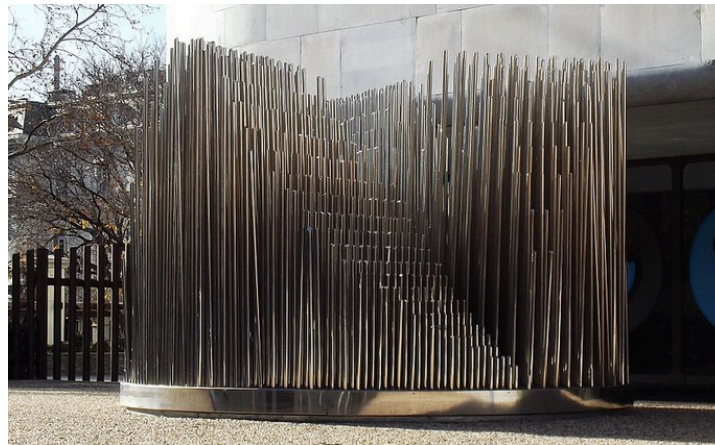


Image 1.b - "Órgano" by sculptor Eusebio Sempere is large-scale example of a phononic crystal

2. Behavior and mechanisms of acoustic metamaterials

As discussed previously, unusual properties of acoustic metamaterials typically arise from the structural properties of the material rather than its composition. Two of these structural properties are bulk modulus and mass density - bulk modulus being a measure of a substance's resistance to uniform compression. For acoustic materials and acoustic metamaterials, both bulk modulus and density are component parameters, which define their refractive index. In certain frequency bands (spectral bands), the effective mass density and bulk modulus may become negative [6] - this results in a negative refractive index which is a property of so called negative-index (or "left-handed") metamaterials.

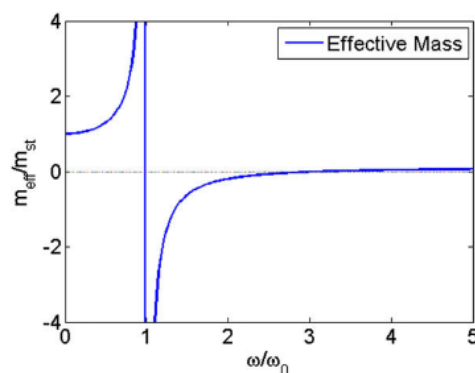


Image 2.a - Effective mass as a function of resonant frequency in a lattice-structure metamaterial

While negative effective mass (density) is a result of shortcomings in how metamaterials are modeled (their mass isn't actually negative), Huang, Sun and Huang [6] have demonstrated that the effective mass in a monatomic metamaterial lattice system must be frequency dependent and allowed to become negative. Simply put, an object with negative effective mass moves in the opposite direction when pushed - or in the contexts of acoustics - cause sound waves to bend in the opposite direction compared to their normal path in traditional materials. Negative effective mass in acoustic metamaterials is primarily achieved by embedding resonant structures which create a phase shift that mimics the effect of negative mass [7]. When sound waves hit the metamaterial, these resonators vibrate in such a way that their effective response creates the appearance of negative mass. At specific frequencies, the resonators are out of phase with the sound wave, leading to the material behaving as if it has negative mass.

The effect of negative bulk modulus was utilized to create a so-called "acoustic double-fishnet" structure [8] in 2006, which can be utilized to provide acoustic shielding to block environmental noise. The "fishnet" consists of a waveguide (a structure that guides waves by restricting the transmission of energy to one direction) loaded with an array of coupled Helmholtz resonators - closed cavities connected to a waveguide via a narrow channel. At their collective resonance frequency, a low-frequency stopband is formed, the origin of which can be traced back to the negative effective bulk modulus. Thus, attaching a series of open side-branches to the waveguide produces resonators with very low resonance frequency, and sound waves are entirely reflected up to the frequency at which the sign of the bulk modulus changes.

The aforementioned "stopband" (otherwise referred to as a band gap) that occurs in low frequency ranges in acoustic metamaterials happens due to local resonances and a phenomenon known as Bragg scattering [9]. Bragg scattering is a fundamental concept in wave physics, particularly in the study of waves interacting with periodic structures. It occurs when waves, such as sound or electromagnetic waves, are scattered by a periodic array of scatterers - such as those found in the structure of metamaterials.

$$n\lambda = 2d \sin \theta$$

Image 2.b - Bragg's law - The condition for constructive interference

Locally resonant band-gaps and Bragg's law cause phononic crystals to forbid the propagation of elastic waves in some frequency ranges, but also to permit them in other ranges - these are analogous to electronic band gaps in semiconductor physics. Band gaps are a feature of acoustic metamaterials which enables them to be applied for noise cancellation, filtering, vibration control and imaging, depending on how the band stops and passes are engineered.

Another useful property of acoustic metamaterials - specifically negative index (left-handed) metamaterials - is their ability of refracting waves negatively. This leads to the technical possibility of metamaterials achieving superlensing—that is, to focus or image beyond the diffraction limit [10] (a term borrowed from optics referring to the limit of resolution performance of a lens). In traditional acoustics, the resolution of a lens is limited by the wavelength of the sound waves (diffraction limit). However, acoustic metamaterials with specially engineered structures can overcome this limitation, enabling subwavelength imaging and focusing, which is the basis of superlensing. Helmholtz resonators, which usually induce a negative bulk modulus, were first used

in the design of a super lens [11] and were designed as a planar network to focus ultrasound in water. Acoustic superlenses constructed from metamaterials have great potential for use in medicine (ultrasound) and more precise sonar systems.



Image 2.c - Example of an ultrasound lens - when filled with water, the holes in this aluminum plate act as resonant cavities that can focus ultrasound waves

The concept of metamaterial “cloaks” to render an object functionally invisible has been explored since the start of the 2000s, initially in electromagnetism and optics [13]. These cloaks would hypothetically function by manipulating the paths traversed by light through a novel optical material. Analogously, acoustic cloaking would make objects impervious to sound waves, which could be used to build soundproof homes, advanced concert halls, or stealth warships. Making such a cloaking metamaterial for a sound means modifying the acoustic analogues to permittivity and permeability in light waves, which are the material's mass density and its elastic constant. The main difficulty in creating such a cloak is in the material design itself - experimental designs suffer from energy losses due to absorption and scattering [10] which in turn reduce its effectiveness especially for sound waves in multiple frequencies. This prevents the cloak from scattering sound waves evenly and cleanly. Current research is exploring ways to develop broadband cloaks by combining different types of resonators or using tunable metamaterials that can adapt to different frequencies. Another avenue of research is reducing the complexity of the applied metamaterials, as well as scaling them down for real world application (i.e. reducing their bulkiness).

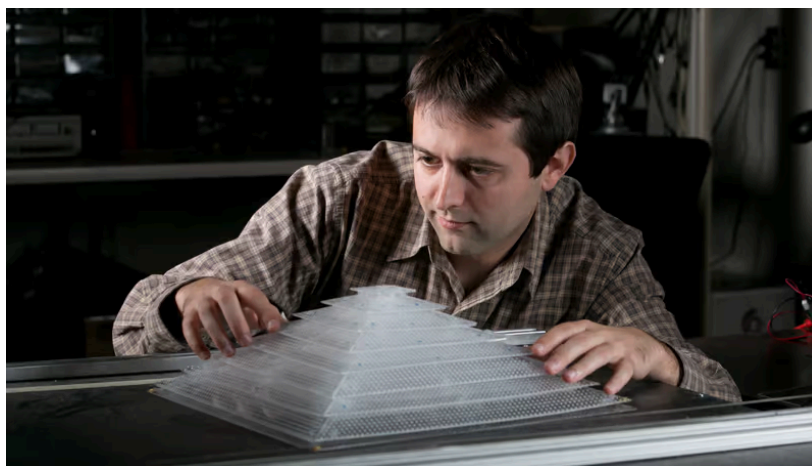


Image 2.d - Research scientist Bogdan Popa with plastic 3D printed prototype acoustic cloak

3. Types, simulation and fabrication

Based on the physical mechanisms discussed in the previous chapter as well as structural designs (architecture) there are different types (or categories) of acoustic metamaterials. Given that research on acoustic metamaterials is a fairly new field there are no strict, defined categories, though some type definitions are more prominent in research than others. One possible categorization of acoustic metamaterials which will be utilized here is based on the characteristics of their structures and the properties of their responses to acoustic waves [12]: **locally resonant structures, Helmholtz resonant structures, membrane-type structures and coiling-up space structures.**

Locally resonant acoustic metamaterials rely on local resonators that interact with sound waves at specific frequencies. The resonators are typically much smaller than the wavelength of the sound, and they vibrate in response to sound waves manipulating their propagation. These resonators can further be classified into unipolar resonance elements and dipole resonance elements. The typical mathematical model of a dipole resonance unit is a two-degree-of-freedom mass-in-mass system, the practical model of which includes the original design of a shot ball covered with rubber, as investigated by Liu et al. [4]

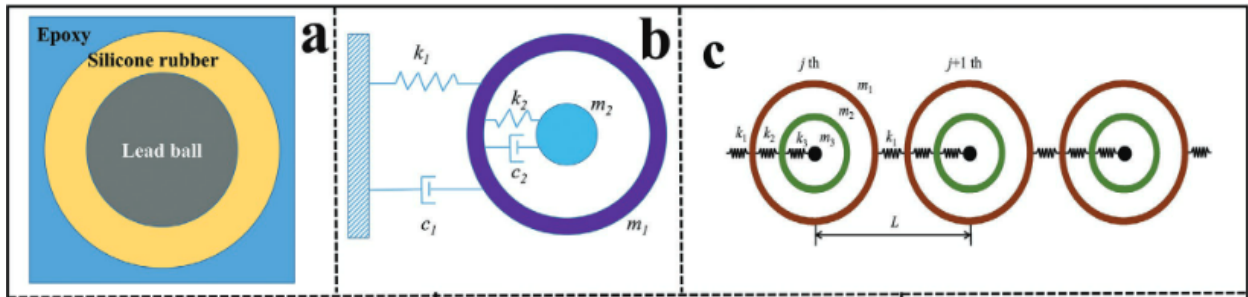


Image 3.a - Some types of locally resonant acoustic metamaterials - (a) Lead spherical cores coated with silicone rubber (mentioned in chapter 1) - (b) Mass-in-mass model embedded with piezoelectric stack. - (c) Mass-in-mass model with damped oscillator

Helmholtz resonant structures - a typical Helmholtz resonator consists of a neck and an air cavity and is generally used in acoustic absorption and noise reduction. Research from 2006 by Fang et al. [14] proposed a type of acoustic metamaterial with a 1D Helmholtz resonant cavity in the subwavelength category and observed that the equivalent elastic modulus was negative when resonance occurred. Further, it has been observed that an open short tube Helmholtz cavity provides an effective low-frequency bandgap. To handle hybrid noise frequencies, a Helmholtz cavity array can be created - resonators are arranged periodically on either side of a pipe with the same cross-sectional area. With this set-up, broadband noise isolation was achieved at the expected resonance frequencies. In addition, the thicknesses of the designed acoustic devices can be optimized for the predicted frequencies.

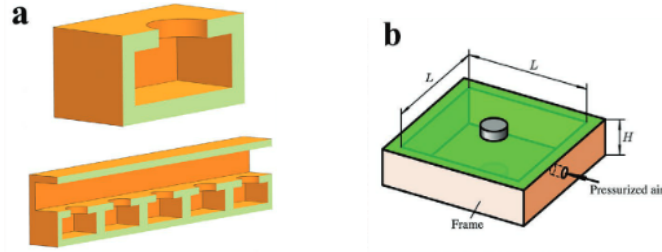


Image 3.b - (a) Helmholtz resonant structures - singular and in an array; (b) Membrane-type structure

Membrane-type structures - this type of acoustic metamaterial consists of a membrane fixed onto a hard boundary, to which a central mass block may be attached. This design generally improves upon structures which exclusively use Helmholtz resonators to achieve sound absorption on multiple frequency bands. While most membrane-type acoustic metamaterials rely on negative density (negative effective mass) the introduction of metamaterials with both negative effective mass and bulk modulus significantly increased the effective bandwidth and reduced the sound transmission loss. An additional improvement introduced to membrane-type metamaterials are noncontact gradient magnetic fields - these magneto-metamaterials achieved dynamic regulation of equivalent density, whereas the sound was completely controlled via a gradient magnetic field.

Coiling-up space structures - of the structures discussed so far, locally resonant metamaterials have limitations with relatively small bandgaps whereas membrane type metamaterials entail problems in terms of short lifespans (easy deformation) and unstable performance. Coiling-up space metamaterials are a class that manipulate waves by coiling or bending the propagation path, thus creating a much longer effective path and controlling their behavior (creating high refractive indices). These metamaterials often have labyrinthine structures created via bending and perforation of materials, as well as coils and spirals introduced to further elongate wave paths. Coiling-up space acoustic metamaterials achieve excellent acoustic properties, such as double negativity, near total reflection and a large acoustic transmission loss. However this type of structure is inherently complex and is difficult to scale down for practical purposes.

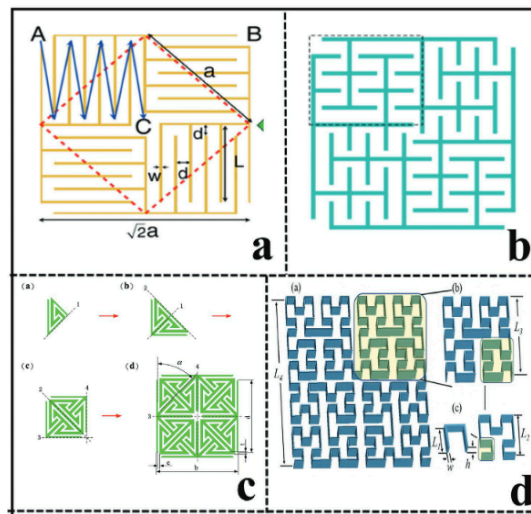


Image 3.c - Various designs for coiling-up space structures

Before applying these various designs in practice, **simulation** is a potent tool in exploring how various types of acoustic metamaterials interact with sound waves. Simulation involves modeling their complex interactions using computational methods. The behavior of sound waves in metamaterials is governed by principles of acoustics, elasticity, and sometimes fluid dynamics, making simulation a multi-physics problem. These simulations help researchers predict the performance of metamaterials before they are fabricated, optimizing their design for specific applications like noise control, vibration isolation, or acoustic cloaking. Parameters such as the overall structural design, the target bandgap, resulting sound transmission loss can be tweaked [15] and estimated using simulations.

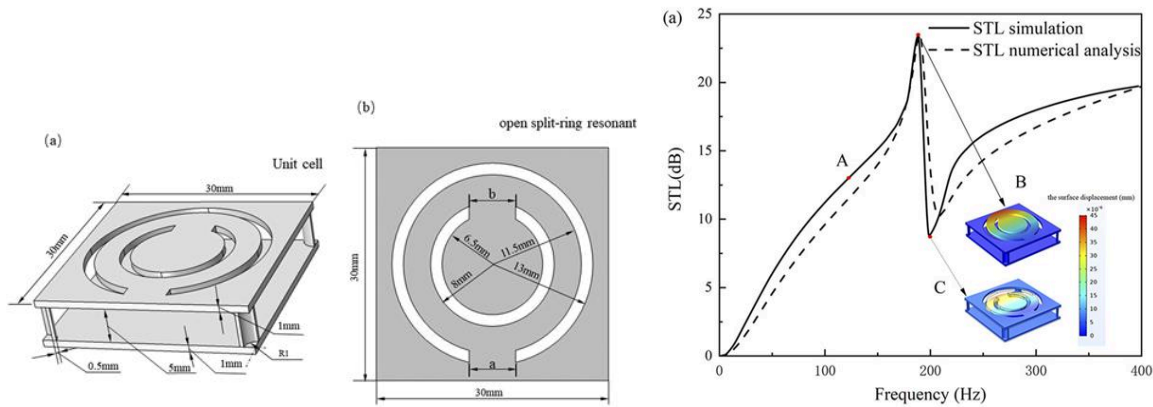


Image 3.d - Examples from research [15] on simulating sound transmission loss (STL) in a novel design

Fabrication for real-world application has presented a significant challenge in acoustic metamaterial research since their introduction [12]. Conventional fabrication methods include laser cutting, CNC machines and assembly from parts - all of which can be used to fabricate several simple 2D (and sometimes 3D) acoustic metamaterials, such as local resonant structures and Helmholtz resonance structures (among others). On the other hand additive fabrication, more commonly known as 3D printing has progressed rapidly in recent years and is now being widely used in numerous fields, including metamaterial manufacture. The emergence of additive manufacturing has greatly facilitated the development and practical application of acoustic metamaterials. FDM (fused deposition modeling), a low-cost, high-speed and simple operation method has become one of the most commonly used additive manufacturing technologies.

While there is significant progress in fabrication for research and limited-scope applications, industrial-scale fabrication still faces significant challenges. Large-scale metamaterials often need to have uniform properties across the entire structure to ensure consistent performance. Achieving uniformity in terms of material density, elasticity, and structural integrity over large areas is difficult, especially with the inclusion of resonating elements like Helmholtz resonators, which need to be precisely tuned. Ensuring the scalability and reproducibility of complex designs for widespread (and commercial) applications is a potent research avenue for the future.

4. Applications

Acoustic metamaterials have a wide range of possible applications due to their ability to manipulate sound waves in unique ways. As mentioned previously, the very nature of metamaterials enables them to exceed the usual limitations of conventional materials. While practical application is in most cases still in its infancy, the potential of acoustic metamaterials is enormous.

Sound absorption and isolation - Noise pollution has been a major issue impacting life for decades now, including traffic noise, loud machinery and the general bustle of a cityscape. Dissipating sound energy and isolating sound propagation are two strategies to reduce or even eliminate noise pollution, however conventional materials are bulky and usually applied in specific cases (such as sound treatment for a studio). Conventional materials are also generally ineffective for strong low frequency waves due to their high penetrative ability and weak attenuation. Acoustic metamaterials developed for sound absorption/isolation are constructed from membrane-type resonators or Helmholtz resonators [16]. By employing a bandgap in these resonant metamaterials their subwavelength structures could efficiently improve sound absorption and block sound propagation, respectively.

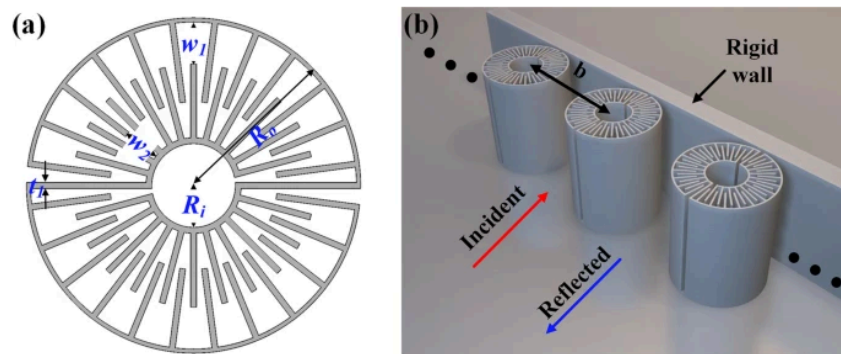


Image 4.a - Proposed low frequency absorptive system ("bass trap") utilizing metamaterials

Resonant features of acoustic metamaterials tend to narrow the bandwidth and limit the absorption spectrum. Hybrid systems composed of multifrequency resonant units with a slight resonance frequency shift between one another can be designed to obtain a better overall absorption spectrum for broad frequencies. Another possible solution are "tunable" - active metamaterials, which are a class of metamaterials whose acoustic properties can be dynamically controlled or adjusted in real time. Unlike traditional, passive metamaterials that have fixed properties once fabricated, tunable or active metamaterials use external stimuli to modify how they interact with sound waves, enabling them to be more adaptive and versatile.

Acoustic lens - In a manner analogous to optical lenses, an acoustic lens focuses sound waves, concentrating acoustic energy at a specific point. Acoustic lenses (and superlensing) are an important research subject of acoustic metamaterials since their inception, given that Sir John Pendry proposed metamaterials as a way to go beyond the diffraction limit of a conventional lens. Since then research on metamaterials has been stimulated by the opportunity to develop artificial media that refracts waves in their negative direction. In fact phononic crystals were the first

development with negative refractive properties for sound waves. Beam steering in phononic crystals can be achieved by Bragg scattering, leading to enhanced diffraction in the negative direction [17], and one of the first experiments featuring phononic crystals involved focusing ultrasound by utilizing negative refraction.

In industrial applications, acoustic lenses are used to focus sound waves for detecting flaws or cracks within materials. By focusing sound energy precisely, acoustic lenses improve the ability to detect imperfections in structures without damaging them. Acoustic lenses can also be used in architectural acoustics, such as in concert halls or auditoriums, to direct sound in specific ways, enhancing the auditory experience for the audience by ensuring even sound distribution throughout the space. Another intriguing application is in **acoustic holography**, which is a technique used to visualize and reconstruct sound fields in three dimensions by capturing the properties of sound waves, such as their amplitude and phase, as they propagate through a medium (e.g., air, water, or solid materials). It is analogous to optical holography, which reconstructs visual images using light waves, but in this case, sound waves are used instead. Acoustic holography is widely used to pinpoint noise sources in environments such as factories, airplanes, or machinery. By visualizing the sound field, engineers can identify the exact locations of noise emissions and take corrective actions to reduce them.

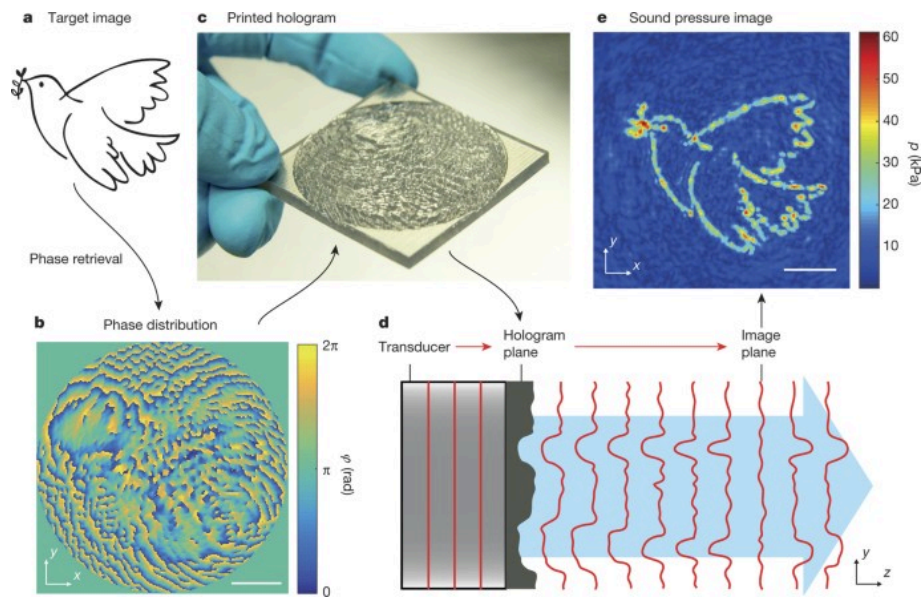


Image 4.b - Example of an acoustic hologram and the resulting sound pressure image

Medical imaging - acoustic metamaterials have potential to enhance ultrasound-based imaging by increasing efficiency, resolution and accuracy. As previously mentioned, conventional ultrasound imaging is limited by the diffraction limit, restricting resolution to about half the wavelength of the sound used. Through the usage of acoustic metamaterials, a significantly greater resolution can be achieved, which allows for the visualization of smaller structures within the body, such as microvasculature or tiny tumors. One experimental demonstration [17] focuses ultrasound waves through a flat acoustic metamaterial lens composed of a planar network of subwavelength

Helmholtz resonators. This enabled the metamaterial lens possess a variable focal length at different frequencies.

Seismic research - there are several emerging applications in seismic research due to acoustic metamaterial development, particularly in areas related to controlling and mitigating the impact of seismic waves. By harnessing their ability to manipulate wave propagation, metamaterials can potentially reduce earthquake damage and improve seismic imaging techniques. Acoustic metamaterials can be used in the design of vibration isolation systems for structures. By embedding metamaterials beneath or around a building's foundation, it is possible to absorb or reflect incoming seismic waves, thus reducing the energy transmitted to the structure. These metamaterial layers can be engineered to block specific low-frequency seismic waves that are most dangerous to buildings.

One research paper [18] explores the low-frequency bandgap characteristics of a proposed soil-based acoustic metamaterial which can suppress the propagation of vibrations and reduce the risk of extreme loadings such as earthquakes. Two types of soil-expanded lattices were created, and the bandgap characteristics of the lattice are obtained through the transfer matrix method. The propagation process for finite periodic lattice is simulated by the finite difference method in the time domain and found that the acceleration amplitudes in the wave propagation region are suppressed by 90% for the seismic metamaterial with rubber components.

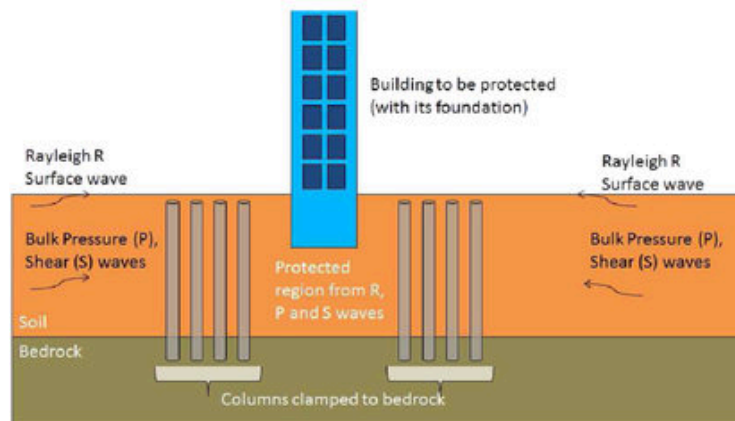


Image 4.c - Proposed metamaterial column design which would protect a building's foundations from various types of destructive waves caused by earthquakes

Musical instruments - musical instruments are complex structures and some may already qualify as metamaterials of a sort. Altering the instrument geometries and adding metamaterial behavior can increase the instruments sound variability and articulatory possibilities or lead to sounds not expected from mechanical instruments at all. One example of a modification [19] altered a frame drum by adding additional point masses forming a ring which caused the frame drum to show cloaking behavior when struck in the middle of the ring, where frequencies within a certain frequency band cannot leave the ring. Still when striking the drum outside the ring a normal drum sound is achieved, giving the drummer a wider range of options for articulation. Changing existing

instrument geometries can lead to added band gaps in their spectrum, and using several of such band gaps will lead to a designed sound.



Image 4.d - Modified frame drum [19] positioned in front of a microphone array

A more general application for musical instruments would be enhanced control and filtering of unwanted noise and frequency bands. This would potentially improve the “purity” of an instrument’s sound and could be particularly useful for controlling resonances and overtones.

Conclusion

Since their inception metamaterials have generated great potential in a broad range of scientific and engineering domains. In the field of acoustics they enable a transformative approach to controlling sound waves which goes beyond the capabilities of conventional materials. By utilizing specifically engineered structures with unique properties such as negative effective mass and bulk modulus these materials open up new possibilities in noise reduction and wave focusing. These properties can consequently be utilized to create unique phenomena such as acoustic cloaking, bandgap filtering and superlensing, as well as to create advancements in a multitude of fields such as medicine, seismic protection, military technology and music.

However real world application of acoustic metamaterials faces many challenges, which have so far inhibited them mostly to research and individual experimental applications. Scalability and cost have been their primary detriment, given how complex some types of metamaterials can be for fabrication, especially on an industrial scale. Future research will no doubt need to look into cheaper and more efficient fabrication methods, perhaps utilizing 3D printing on a larger scale to reduce production costs.

As research continues, tunable and active metamaterials with adaptive characteristics will surely be one important avenue for progress, given that they can bypass common pitfalls of metamaterials designed for a single purpose or scenario. The interdisciplinary nature of acoustic metamaterial research which bridges physics, material engineering and all of their possible fields of application will ensure that they remain at the forefront of innovation in the coming years.

Citations

- [1] Craster, R. V., & Guenneau, S. (Eds.). (2013). *Acoustic Metamaterials*. Springer Series in Materials Science
- [2] Ziolkowski, R. W. & Engheta, N. (Eds.). (2006). *Metamaterials: Physics and Engineering Explorations*. Wiley-IEEE Press
- [3] Kinsler, L. E., Frey, A. R., Coppens, A. B., & Sanders, J. V. (2000). *Fundamentals of acoustics*. John Wiley & sons.
- [4] Liu, Zh., Zhang, X., Mao Y., Zhu, Y. Y., Yang, Zh., Chan, C. T., Sheng, P. (2000). Locally Resonant Sonic Materials. *Science*.
- [5] Srivastava, G. P. (2022). *The Physics of Phonons*. United Kingdom: CRC Press.
- [6] Huang, H.H. & Sun, C.T. & Huang, G.L.. (2009). On the negative effective mass density in acoustic metamaterials. *International Journal of Engineering Science*.
- [7] Rodrigues, Inês & Steele, Gary & Bothner, Daniel. (2022). Photon-Pressure with a Negative Mass Microwave Mode.
- [8] Cummer, S.A., Christensen, J., & Alú, A. (2016). Controlling sound with acoustic metamaterials. *Nature Reviews Materials*, 1, 16001.
- [9] Wen, Shurui & Xiong, Yuanhao & Hao, Shuaimin & Li, Fengming & Zhang, Ch. (2019). Enhanced band-gap properties of an acoustic metamaterial beam with periodically variable cross-sections. *International Journal of Mechanical Sciences*.
- [10] Kaina, Nadege & Lemoult, Fabrice & Fink, Mathias & Lerosey, Geoffroy. (2015). Negative refractive index and acoustic superlens from multiple scattering in single negative metamaterials. *Nature*.
- [11] Zhang, S., Yin, L., & Fang, N. (2009). Focusing ultrasound with an acoustic metamaterial network. *Physical review letters*, 102(19), 194301.
- [12] Liao, G., Luan, C., Wang, Z., Liu, J., Yao, X., & Fu, J. (2021). *Acoustic Metamaterials: A Review of Theories, Structures, Fabrication Approaches, and Applications*. *Advanced Materials Technologies*, 6.
- [13] D. Schurig *et al.* (2006). Metamaterial Electromagnetic Cloak at Microwave Frequencies. *Science* **314**,977-980
- [14] Fang, N., Xi, D., Xu, J., Ambati, M., Srituravanich, W., Sun, C., & Zhang, X. (2006). Ultrasonic metamaterials with negative modulus. *Nature materials*, 5(6), 452–456.
- [15] Zhang, Jiangyi & Hu, Bo & Wang, Shibo. (2023). Review and perspective on acoustic metamaterials: From fundamentals to applications. *Applied Physics Letters*.
- [16] Lu, Qiangbing & Zhang, Xiujuan & Lu, Minghui & Chen, Yanfeng. (2022). Perspective: Acoustic Metamaterials in Future Engineering. *Engineering*. 17. 10.1016/j.eng.2022.04.020.
- [17] Zhang, Shu & Yin, Leilei & Fang, Nicholas. (2009). Focusing Ultrasound with an Acoustic Metamaterial Network. *Physical review letters*. 102. 194301. 10.1103/PhysRevLett.102.194301.
- [18] Bai, Yongtao & Li, Xiaolei & Zhou, Xuhong & Li, Peng & Beer, Michael. (2023). Soil-expanded seismic metamaterial with ultralow and wide bandgap. *Mechanics of Materials*. 180. 104601. 10.1016/j.mechmat.2023.104601.
- [19] Bader, Rolf & Fischer, Jost & Münster, Malte & Kontopidis, Patrick. (2019). Metamaterials in musical acoustics: A modified frame drum. *The Journal of the Acoustical Society of America*. 145. 3086-3094. 10.1121/1.5102168.